a. The electric field is inversely proportional to the density of electric field lines.
b. The electric field is directly proportional to the density of electric field lines.
c. The electric field is not related to the density of electric field lines.
d. The electric field is inversely proportional to the square root of density of electric field lines.

18. If five electric-field lines come out of a +5 nC charge, how many electric-field lines should come out of a +20 nC charge?
a. five field lines
b. 10 field lines
c. 15 field lines
d. 20 field lines

18.4 Electric Potential

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the similarities and differences between electric potential energy and gravitational potential energy
- Calculate the electric potential difference between two point charges and in a uniform electric field

Section Key Terms

electric potential	electric potential energy

As you learned in studying gravity, a mass in a gravitational field has potential energy, which means it has the potential to accelerate and thereby increase its kinetic energy. This kinetic energy can be used to do work. For example, imagine you want to use a stone to pound a nail into a piece of wood. You first lift the stone high above the nail, which increases the potential energy of the stone-Earth system—because Earth is so large, it does not move, so we usually shorten this by saying simply that the potential energy of the stone increases. When you drop the stone, gravity converts the potential energy into kinetic energy. When the stone hits the nail, it does work by pounding the nail into the wood. The gravitational potential energy is the work that a mass can potentially do by virtue of its position in a gravitational field. Potential energy is a very useful concept, because it can be used with conservation of energy to calculate the motion of masses in a gravitational field.

Electric potential energy works much the same way, but it is based on the electric field instead of the gravitational field. By virtue of its position in an electric field, a charge has an electric potential energy. If the charge is free to move, the force due to the electric field causes it to accelerate, so its potential energy is converted to kinetic energy, just like a mass that falls in a gravitational field. This kinetic energy can be used to do work. The electric potential energy is the work that a charge can do by virtue of its position in an electric field.

The analogy between gravitational potential energy and electric potential energy is depicted in Figure 18.21. On the left, the ball-Earth system gains gravitational potential energy when the ball is higher in Earth's gravitational field. On the right, the two-charge system gains electric potential energy when the positive charge is farther from the negative charge.

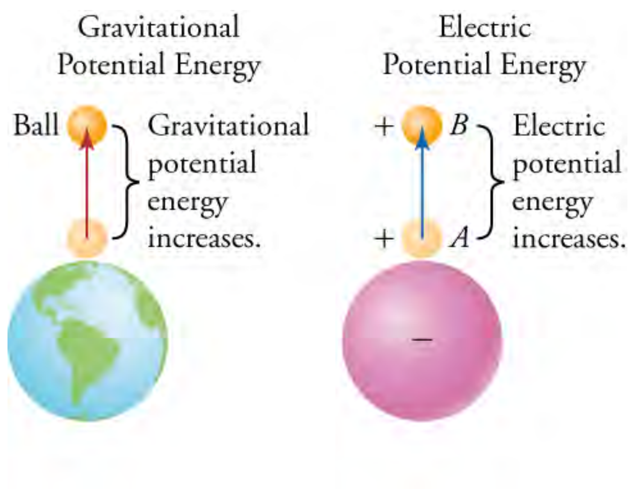


Figure 18.21 On the left, the gravitational field points toward Earth. The higher the ball is in the gravitational field, the higher the potential energy is of the Earth-ball system. On the right, the electric field points toward a negative charge. The farther the positive charge is from the negative charge, the higher the potential energy is of the two-charge system.

Let's use the symbol U_G to denote gravitational potential energy. When a mass falls in a gravitational field, its gravitational potential energy decreases. Conservation of energy tells us that the work done by the gravitational field to make the mass accelerate must equal the loss of potential energy of the mass. If we use the symbol $W_{\text{donebygravity}}$ to denote this work, then

$$-\Delta U_{\text{G}} = W_{\text{donebygravity}}, \tag{18.18}$$

where the minus sign reflects the fact that the potential energy of the ball decreases.

The work done by gravity on the mass is

$$W_{\text{donebygravity}} = -F(y_{\text{f}} - y_{\text{i}}\ ), \tag{18.19}$$

where F is the force due to gravity, and y_{i} and y_{f} are the initial and final positions of the ball, respectively. The negative sign is because gravity points down, which we consider to be the negative direction. For the *constant* gravitational field near Earth's surface, $F = mg$. The change in gravitational potential energy of the mass is

$$-\Delta U_{\text{G}} = W_{\text{donebygravity}} = -F\left(y_{\text{f}} - y_{\text{i}}\right) = -mg\left(y_{\text{f}} - y_{\text{i}}\right), \text{or} \Delta U_{\text{G}} = mg\left(y_{\text{f}} - y_{\text{i}}\right). \tag{18.20}$$

Note that $y_{\text{f}} - y_{\text{i}}$ is just the negative of the height h from which the mass falls, so we usually just write $\Delta U_{\text{G}} = -mgh$.

We now apply the same reasoning to a charge in an electric field to find the electric potential energy. The change ΔU_{E} in electric potential energy is the work done by the electric field to move a charge q from an initial position x_{i} to a final position x_{f} ($-\Delta U_{\text{E}} = W_{\text{donebyE-field}}$). The definition of work does not change, except that now the work is done by the electric field: $W_{\text{donebyE-field}} = F(x_{\text{f}} - x_{\text{i}})$. For a charge that falls through a *constant* electric field E, the force applied to the charge by the electric field is $F = qE$. The change in electric potential energy of the charge is thus

$$-\Delta U_{\text{E}} = W_{\text{donebyE-field}} = Fd = qE\left(x_{\text{f}} - x_{\text{i}}\right) \tag{18.21}$$

or

$$\Delta U_{\text{E}} = -qE\left(x_{\text{f}} - x_{\text{i}}\right). \tag{18.22}$$

This equation gives the change in electric potential energy of a charge q when it moves from position x_{i} to position x_{f} in a *constant* electric field E.

Figure 18.22 shows how this analogy would work if we were close to Earth's surface, where gravity is constant. The top image shows a charge accelerating due to a constant electric field. Likewise, the round mass in the bottom image accelerates due to a constant gravitation field. In both cases, the potential energy of the particle decreases, and its kinetic energy increases.

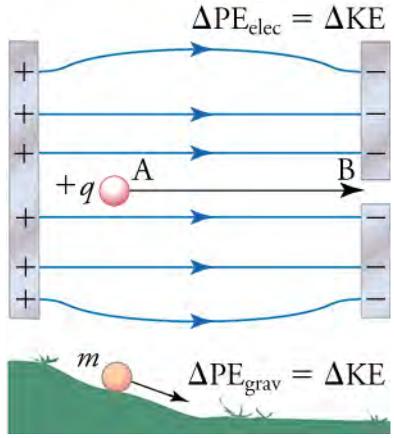


Figure 18.22 In the top picture, a mass accelerates due to a constant electric field. In the bottom picture, the mass accelerates due to a constant gravitational field.

WATCH PHYSICS

Analogy between Gravity and Electricity

This video discusses the analogy between gravitational potential energy and electric potential energy. It reviews the concepts of work and potential energy and shows the connection between a mass in a uniform gravitation field, such as on Earth's surface, and an electric charge in a uniform electric field.

Click to view content (https://www.openstax.org/l/28grav-elec)

If the electric field is not constant, then the equation $\Delta U_{\mathrm{E}} = -qE\left(x_{\mathrm{f}} - x_{\mathrm{i}}\right)$ is not valid, and deriving the electric potential energy becomes more involved. For example, consider the electric potential energy of an assembly of two point charges q_1 and q_2 of the same sign that are initially very far apart. We start by placing charge q_1 at the origin of our coordinate system. This takes no electrical energy, because there is no electric field at the origin (because charge q_2 is very far away). We then bring charge q_2 in from very far away to a distance *r* from the center of charge q_1. This requires some effort, because the electric field of charge q_1 applies a repulsive force on charge q_2. The energy it takes to assemble these two charges can be recuperated if we let them fly apart again. Thus, the charges have potential energy when they are a distance *r* apart. It turns out that the electric potential energy of a pair of point charges q_1 and q_2 a distance *r* apart is

$$U_{\mathrm{E}} = \frac{kq_1q_2}{r} \qquad (18.23)$$

To recap, if charges q_1 and q_2 are free to move, they can accumulate kinetic energy by flying apart, and this kinetic energy can be used to do work. The maximum amount of work the two charges can do (if they fly infinitely far from each other) is given by the equation above.

Notice that if the two charges have opposite signs, then the potential energy is negative. This means that the charges have more potential to do work when they are *far* apart than when they are at a distance *r* apart. This makes sense: Opposite charges attract, so the charges can gain more kinetic energy if they attract each other from far away than if they start at only a short distance apart. Thus, they have more potential to do work when they are far apart. Figure 18.23 summarizes how the electric potential energy depends on charge and separation.

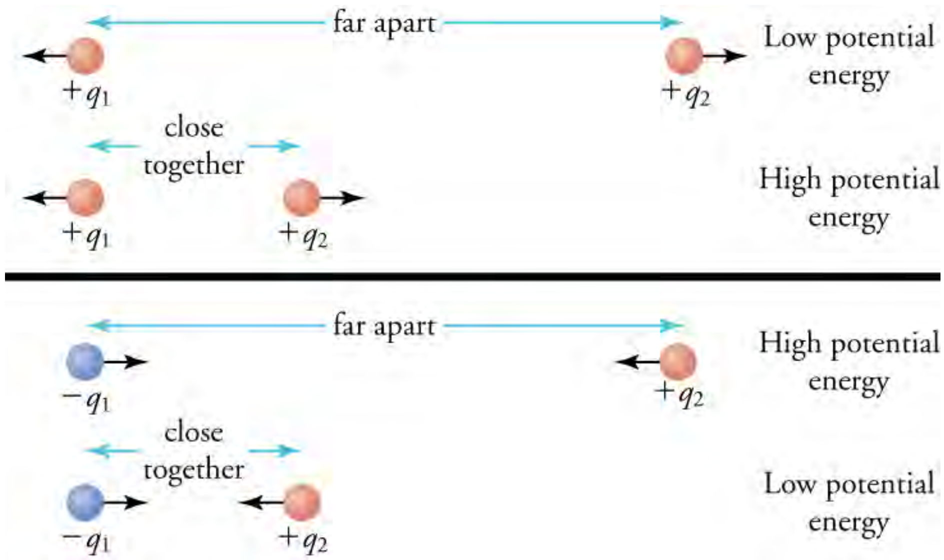


Figure 18.23 The potential energy depends on the sign of the charges and their separation. The arrows on the charges indicate the direction in which the charges would move if released. When charges with the same sign are far apart, their potential energy is low, as shown in the top panel for two positive charges. The situation is the reverse for charges of opposite signs, as shown in the bottom panel.

Electric Potential

Recall that to find the force applied by a fixed charge Q on any arbitrary test charge q, it was convenient to define the electric field, which is the force per unit charge applied by Q on any test charge that we place in its electric field. The same strategy is used here with electric potential energy: We now define the **electric potential** V, which is the electric potential energy per unit charge.

$$V = \frac{U_\mathrm{E}}{q} \tag{18.24}$$

Normally, the electric potential is simply called the *potential* or *voltage*. The units for the potential are J/C, which are given the name *volt* (V) after the Italian physicist Alessandro Volta (1745–1827). From the equation $U_\mathrm{E} = kq_1q_2/r$, the electric potential a distance r from a point charge q_1 is

$$V = \frac{U_\mathrm{E}}{q_2} = \frac{kq_1}{r}. \tag{18.25}$$

This equation gives the energy required per unit charge to bring a charge q_2 from infinity to a distance r from a point charge q_1. Mathematically, this is written as

$$V = \left.\frac{U_E}{q_2}\right|_{R=r} - \left.\frac{U_E}{q_2}\right|_{R=\infty}. \tag{18.26}$$

Note that this equation actually represents a *difference* in electric potential. However, because the second term is zero, it is normally not written, and we speak of the electric potential instead of the electric potential difference, or we just say the potential difference, or voltage). Below, when we consider the electric potential energy per unit charge between two points not infinitely far apart, we speak of electric potential *difference* explicitly. Just remember that electric potential and electric potential difference are really the same thing; the former is used just when the electric potential energy is zero in either the initial or final charge configuration.

Coming back now to the electric potential a distance r from a point charge q_1, note that q_1 can be any arbitrary point charge, so we can drop the subscripts and simply write

$$V = \frac{kq}{r}. \tag{18.27}$$

Now consider the electric potential near a group of charges q_1, q_2, and q_3, as drawn in Figure 18.24. The electric potential is derived by considering the electric field. Electric fields follow the principle of superposition and can be simply added together, so the electric potential from different charges also add together. Thus, the electric potential of a point near a group of charges is

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \cdots. \tag{18.28}$$

where $r_1, r_2, r_3, \ldots$, are the distances from the center of charges $q_1, q_2, q_3, \ldots$ to the point of interest, as shown in Figure 18.24.

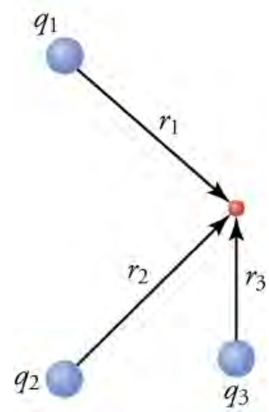


Figure 18.24 The potential at the red point is simply the sum of the potentials due to each individual charge.

Now let's consider the electric potential in a uniform electric field. From the equation $\Delta U_E = -qE(x_f - x_i)$, we see that the potential difference in going from x_i to x_f in a uniform electric field E is

$$\Delta V = \frac{\Delta U_E}{q} = -E(x_f - x_i). \tag{18.29}$$

TIPS FOR SUCCESS

Notice from the equation $\Delta V = -E(x_f - x_i)$ that the electric field can be written as

$$E = \frac{\Delta V}{x_f - x_i} \tag{18.30}$$

which means that the electric field has units of V/m. Thus, if you know the potential difference between two points, calculating the electric field is very simple—you simply divide the potential difference by the distance!

Notice that a positive charge in a region with high potential will experience a force pushing it toward regions of lower potential. In this sense, potential is like pressure for fluids. Imagine a pipe containing fluid, with the fluid at one end of the pipe under high pressure and the fluid at the other end of the pipe under low pressure. If nothing prevents the fluid from flowing, it will flow from the high-pressure end to the low-pressure end. Likewise, a positive charge that is free to move will move from a region with high potential to a region with lower potential.

WATCH PHYSICS

Voltage

This video starts from electric potential energy and explains how this is related to electric potential (or voltage). The lecturer calculates the electric potential created by a uniform electric field.

Click to view content (https://www.openstax.org/l/28voltage)

GRASP CHECK

What is the voltage difference between the positions $x_f = 11$ m and $x_i = 5.0$ m in an electric field of $(2.0\ \text{N/C})\,\hat{x}$?

a. 6 V

b. 12 V
c. 24 V
d. 32 V

LINKS TO PHYSICS

Electric Animals

Many animals generate and/or detect electric fields. This is useful for activities such as hunting, defense, navigation, communication, and mating. Because salt water is a relatively good conductor, electric fish have evolved in all the world's oceans. These fish have intrigued humans since the earliest times. In the nineteenth century, parties were even organized where the main attraction was getting a jolt from an electric fish! Scientists also studied electric fish to learn about electricity. Alessandro Volta based his research that led to batteries in 1799 on electric fish. He even referred to batteries as artificial electric organs, because he saw them as imitations of the electric organs of electric fish.

Animals that generate electricity are called *electrogenic* and those that detect electric fields are called *electroreceptive*. Most fish that are electrogenic are also electroreceptive. One of the most well-known electric fish is the electric eel (see Figure 18.25), which is both electrogenic and electroreceptive. These fish have three pairs of organs that produce the electric charge: the main organ, Hunter's organ, and Sach's organ. Together, these organs account for more than 80percent of the fish's body.

Electric eels can produce electric discharges of much greater voltage than what you would get from a standard wall socket. These discharges can stun or even kill their prey. They also use low-intensity discharges to navigate. The electric fields they generate reflect off nearby obstacles or animals and are then detected by electroreceptors in the eel's skin. The three organs that produce electricity contain electrolytes, which are substances that ionize when dissolved in water (or other liquids). An ionized atom or molecule is one that has lost or gained at least one electron, so it carries a net charge. Thus, a liquid solution containing an electrolyte conducts electricity, because the ions in the solution can move if an electric field is applied.

To produce large discharges, the main organ is used. It contains approximately 6,000 rows of electroplaques connected in a long chain. Connected this way, the voltage between electroplaques adds up, creating a large final voltage. Each electroplaque consists of a column of cells controlled by an excitor nerve. When triggered by the excitor nerve, the electroplaques allow ionized sodium to flow through them, creating a potential difference between electroplaques. These potentials add up, and a large current can flow through the electrolyte.

This geometry is reflected in batteries, which also use stacks of plates to produce larger potential differences.

Figure 18.25 An electric eel in its natural environment. (credit: Steven G. Johnson)

GRASP CHECK

If an electric eel produces 1,000 V, which voltage is produced by each electroplaque in the main organ?

a. 0.17 mV
b. 1.7 mV
c. 17 mV
d. 170 mV

WORKED EXAMPLE

X-ray Tube

Dentists use X-rays to image their patients' teeth and bones. The X-ray tubes that generate X-rays contain an electron source separated by about 10 cm from a metallic target. The electrons are accelerated from the source to the target by a uniform electric field with a magnitude of about 100 kN/C, as drawn in Figure 18.26. When the electrons hit the target, X-rays are produced. (a) What is the potential difference between the electron source and the metallic target? (b) What is the kinetic energy of the electrons when they reach the target, assuming that the electrons start at rest?

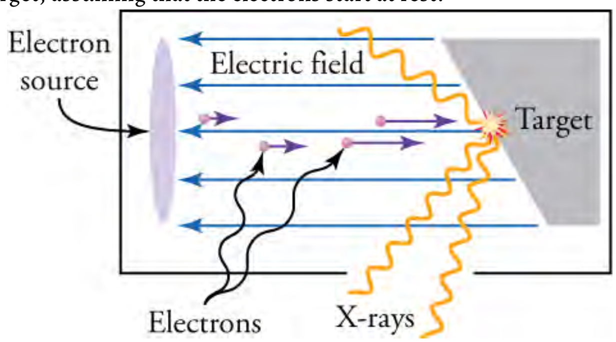


Figure 18.26 In an X-ray tube, a large current flows through the electron source, causing electrons to be ejected from the electron source. The ejected electrons are accelerated toward the target by the electric field. When they strike the target, X-rays are produced.

STRATEGY FOR (A)

Use the equation $\Delta V = -E\,(x_\mathrm{f} - x_\mathrm{i})$ to find the potential difference given a constant electric field. Define the source position as $x_\mathrm{i} = 0$ and the target position as $x_\mathrm{f} = 10\text{ cm}$. To accelerate the electrons in the positive *x* direction, the electric field must point in the negative *x* direction. This way, the force $F = qE$ on the electrons will point in the positive *x* direction, because both *q* and *E* are negative. Thus, $E = -100 \times 10^3\,\text{N/C}$.

Solution for (a)

Using $x_\mathrm{i} = 0$ and $x_\mathrm{f} = 10\text{ cm} = 0.10\text{ m}$, the equation $\Delta V = -E\,(x_\mathrm{f} - x_\mathrm{i})$ tells us that the potential difference between the electron source and the target is

$$\Delta V = -E\,(x_\mathrm{f} - x_\mathrm{i}) = -\left(-100 \times 10^3\,\text{N/C}\right)(0.10\text{ m} - 0) = +10\text{ kV}. \quad (18.31)$$

Discussion for (a)

The potential difference is positive, so the energy per unit positive charge is higher at the target than at the source. This means that free positive charges would fall from the target to the source. However, electrons are negative charges, so they accelerate from the source toward the target, gaining kinetic energy as they go.

STRATEGY FOR (B)

Apply conservation of energy to find the final kinetic energy of the electrons. In going from the source to the target, the change in electric potential energy plus the change in kinetic energy of the electrons must be zero, so $\Delta U_E + \Delta K = 0$. The change in electric potential energy for moving through a constant electric field is given by the equation

$$\Delta U_\mathrm{E} = -qE\,(x_\mathrm{f} - x_\mathrm{i}),$$

where the electric field is $E = -100 \times 10^3\,\text{N/C}$. Because the electrons start at rest, their initial kinetic energy is zero. Thus, the change in kinetic energy is simply their final kinetic energy, so $\Delta K = K_\mathrm{f}$.

Solution for (b)

Again $x_\mathrm{i} = 0$ and $x_\mathrm{f} = 10\text{ cm} = 0.10\text{ m}$. The charge of an electron is $q = -1.602 \times 10^{-19}\,\text{C}$. Conservation of energy gives

$$\begin{aligned} \Delta U_E + \Delta K &= 0. \\ -qE\,(x_\mathrm{f} - x_\mathrm{i}) + K_\mathrm{f} &= 0. \\ K_\mathrm{f} &= qE(x_\mathrm{f} - x_{\mathrm{i.}}). \end{aligned} \quad (18.32)$$

Inserting the known values into the right-hand side of this equation gives

$$\begin{aligned} K_{\mathrm{f}} &= \left(-1.60 \times 10^{-19}\mathrm{C}\right)\left(-100 \times 10^{3}\mathrm{N/C}\right)(0.10\ \mathrm{m} - 0) \\ &= 1.6 \times 10^{-15}\mathrm{J}. \end{aligned} \tag{18.33}$$

Discussion for (b)

This is a very small energy. However, electrons are very small, so they are easy to accelerate, and this energy is enough to make an electron go extremely fast. You can find their speed by using the definition of kinetic energy, $K = \frac{1}{2}mv^2$. The result is that the electrons are moving at more than 100 million miles per hour!

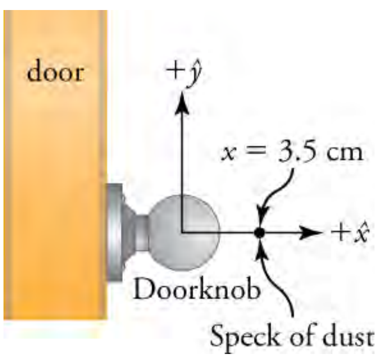
WORKED EXAMPLE

Electric Potential Energy of Doorknob and Dust Speck

Consider again the doorknob from the example in the previous section. The doorknob is treated as a spherical conductor with a uniform static charge $q_1 = -1.5\ \mathrm{nC}$ on its surface. What is the electric potential energy between the doorknob and a speck of dust carrying a charge $q_2 = 0.20\ \mathrm{nC}$ at 1.0 cm from the front surface of the doorknob? The diameter of the doorknob is 5.0 cm.

STRATEGY

As we did in the previous section, we treat the charge as if it were concentrated at the center of the doorknob. Again, as you will be able to validate in later physics classes, we can make this simplification, because the charge is uniformly distributed over the surface of the spherical object. Make a sketch of the situation and define a coordinate system, as shown in the image below. We use $+x$ to indicate the outward direction perpendicular to the door, with $x = 0$ at the center of the doorknob. If the diameter of the doorknob is 5.0 cm, its radius is 2.5 cm. Thus, the speck of dust 1.0 cm from the surface of the doorknob is a distance $r = 2.5\ \mathrm{cm} + 1.0\ \mathrm{cm} = 3.5\ \mathrm{cm}$ from the center of the doorknob. To solve this problem, use the equation $U_{\mathrm{E}} = kq_1q_2/r$.



Solution

The charge on the doorknob is $q_1 = -1.5\ \mathrm{nC} = -1.5 \times 10^{-9}\mathrm{C}$, and the charge on the speck of dust is $q_2 = 0.20\ \mathrm{nC} = 2.0 \times 10^{-10}\mathrm{C}$. The distance $r = 3.5\ \mathrm{cm} = 0.035\ \mathrm{m}$. Inserting these values into the equation $U_{\mathrm{E}} = kq_1q_2/r$ gives

$$\begin{aligned} U_E &= \frac{kq_1q_2}{r} \\ &= \frac{\left(8.99 \times 10^{9}\mathrm{N{\cdot}m^2/C^2}\right)\left(-1.5 \times 10^{-9}\mathrm{C}\right)\left(2.0 \times 10^{-10}\mathrm{C}\right)}{(0.035\ \mathrm{m})} \\ &= -7.7 \times 10^{-8}\mathrm{J}. \end{aligned} \tag{18.34}$$

Discussion

The energy is negative, which means that the energy will decrease that is, get even *more* negative as the speck of dust approaches the doorknob. This helps explain why dust accumulates on objects that carry a static charge. However, note that insulators normally collect more static charge than conductors, because any charge that accumulates on insulators cannot move about on the insulator to find a way to escape. They must simply wait to be removed by some passing moist speck of dust or other host.

Practice Problems

19. What is the electric potential 10 cm from a –10 nC charge?

a. 9.0×10^2 V
b. 9.0×10^3 V
c. 9.0×10^4 V
d. 9.0×10^5 V

20. An electron accelerates from 0 to 10×10^4 m/s in an electric field. Through what potential difference did the electron travel? The mass of an electron is 9.11×10^{-31} kg, and its charge is -1.60×10^{-19} C.

a. 29 mV
b. 290 mV
c. 2,900 mV
d. 29 V

Check Your Understanding

21. Gravitational potential energy is the $(-10\ \mathrm{N/C})\hat{x}$ potential for two masses to do work by virtue of their positions with respect to each other. What is the analogous definition of electric potential energy?

a. Electric potential energy is the potential for two charges to do work by virtue of their positions with respect to the origin point.
b. Electric potential energy is the potential for two charges to do work by virtue of their positions with respect to infinity.
c. Electric potential energy is the potential for two charges to do work by virtue of their positions with respect to each other.
d. Electric potential energy is the potential for single charges to do work by virtue of their positions with respect to their final positions.

22. A negative charge is 10 m from a positive charge. Where would you have to move the negative charge to increase the potential energy of the system?

a. The negative charge should be moved closer to the positive charge.
b. The negative charge should be moved farther away from the positive charge.
c. The negative charge should be moved to infinity.
d. The negative charge should be placed just next to the positive charge.

18.5 Capacitors and Dielectrics

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Calculate the energy stored in a charged capacitor and the capacitance of a capacitor
- Explain the properties of capacitors and dielectrics

Section Key Terms

capacitor	dielectric

Capacitors

Consider again the X-ray tube discussed in the previous sample problem. How can a uniform electric field be produced? A single positive charge produces an electric field that points away from it, as in . This field is not uniform, because the space between the lines increases as you move away from the charge. However, if we combine a positive and a negative charge, we obtain the electric field shown in (a). Notice that, between the charges, the electric field lines are more equally spaced.

What happens if we place, say, five positive charges in a line across from five negative charges, as in Figure 18.27? Now the region between the lines of charge contains a fairly uniform electric field.